A Preliminary Investigation on Bernoulli Models for Battery Energy Storage Systems with State-Dependent Transition Probabilities

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This paper proposes the application of a stochastic model for single-buffer systems with restart level and state-dependent transition probabilities to a Battery Energy Storage System (BESS) in a smart grid. The model represents the charging and discharging processes of a BESS, taking into account the uncertainty in renewable generation and load demand. BESS is treated as a buffer between the upstream renewable source and the downstream load, with state-dependent charging and discharging efficiencies and a specific restart level which controls the input energy flow and, consequently, the system performance. The model is solved analytically using an isolation technique, allowing the derivation of a closed-form solution for the probability distribution of the system states.

Key words: Stochastic modeling; State-dependent transition probabilities; Markov processes; Battery

1. Introduction

The increasing penetration of renewable energy sources in modern power systems has led to new challenges in terms of grid stability and energy management (Zhao et al. 2012). The intermittent and unpredictable nature of solar and wind generation requires the adoption of flexible resources, such as energy storage systems, to balance supply and demand in real time (Fernandez-Blanco et al. 2017). In this context, the management of Battery Energy Storage Systems (BESS) has become a crucial task, complicated by the inherent uncertainty in renewable generation and load demand, making precise scheduling of charging and discharging cycles difficult (Ghiassi-Farrokhfal et al. 2016). Several approaches have been proposed in the literature to address this problem, ranging from deterministic optimization methods (Wu et al. 2014) to stochastic dynamic programming (Zhang et al. 2013). However, most of these techniques rely on simplified battery models and do not fully capture the complex dynamics of the storage process, such as the dependence of charging and discharging efficiencies on the state of charge (SOC) (Rao et al. 2005).

In this paper, we propose a stochastic model for the BESS that accounts for uncertainty in renewable generation and load demand. The proposed model represents the charging and discharging processes of the BESS as a buffer between the upstream renewable source and the downstream load, with state-dependent charging and discharging efficiencies. A key feature of the model is the inclusion of a restart level, which serves to control the input energy flow. By setting a minimum state of charge threshold for the battery to start charging again, the model aims to reduce the probability of charging the BESS with low efficiency values, thus increasing the overall system efficiency and battery shelf life. In fact, a BESS system based on lithium technology is known to work well when charged between 20% and 80% of the charge level Zhang and Zhang (2021). This detailed description of the system is based on a discrete-time Markov process with state-dependent transition probabilities which is solved analytically using an isolation technique.

2. Modeling approach

This section aims to model a stochastic system consisting of a BESS, i.e. the buffer, which receives discrete unit amounts of energy from an upstream charging process and releases discrete unit amounts of energy to a downstream discharging process. Specifically,

- the energy flow is modeled as a discrete flow of unit amounts of energy (e.g., 1 unit amount of energy can be equal to 1 Ah);
- the input probability that one unit amount of energy enters the BESS (from the upstream process) depends on the charge level, i.e., the input probability is state-dependent;
- the output probability that one unit amount of energy leaves the BESS (to the downstream process) depends on the charge level, i.e., the output probability is state-dependent;
- if the BESS is empty no output flow is allowed (the discharging process is starved);
- if the BESS is fully charged no input flow is allowed (the charging process is blocked);
- once the BESS gets fully charged, only the discharging flow is allowed until a certain threshold level (called “restart level” in the sequel) is reached: the charge level can only decrease or remain constant in this phase;
- once the restart level is reached, the charging process is allowed to restart: the charge level can increase/decrease/remain constant according to the stochastic dynamics of the system.

Since charging flow is not always allowed, it is possible to identify two different system behaviors that we call *standard operation* and *energy drainage*. In the *standard operation* behavior both the charging and the discharging processes are allowed: the energy storage receives one unit amount of energy in a time unit with probability $p_1^{(i)}$, being i the current charge level; it releases one unit amount of energy in a time unit with probability $p_2^{(i)}$, being i the current charge level. In the *energy drainage* behavior only the discharging process is allowed while the charging process is interrupted: the charge level i decreases with probability $p_2^{(i)}$ or remains constant with probability (1-$p_2^{(i)}$). The switch from the *standard operation* behavior to the *energy drainage* behavior occurs when the BESS is fully charged (the charge level i is equal to the energy storage capacity N); the switch back from the *energy drainage* behavior to the *standard operation* behavior occurs when the charge level reaches the restart level, denoted as L in the sequel. Hence, for charge levels below the restart level $(i < L)$ both flows are allowed (the only feasible behavior is the *standard operation* behavior); for charge levels above the restart level $(i \geq L)$, we have to distinguish which of the two behaviors the system is following.

We assume here that the system states coincide with the charge levels, by distinguishing between the two behaviors when we are above the restart level L. Hence, it is convenient to partition the state space into three partitions:

- *Standard operation A* partition: it consists of L states $(0, 1, \ldots, L-1)$ and represents the system below the restart level, where the only feasible behavior is the *standard operation* behavior;
- *Standard operation B* partition: it consists of $N - L + 1$ states $(L, L+1, \ldots, N-1, N)$ and represents the system at the restart level or above the threshold level, given that the system is following the *standard operation* behavior (both flows are allowed);

• *Energy drainage* partition: it consists of $N-L-1$ states $(L+1,\dots,N-1)$ and represents the system above the restart level given that the system is following the *energy drainage* behavior (the charge level is decreasing after having reached the full state).

Given the three aforementioned state partitions, the so-called *isolation technique* can be applied in order to simplify the mathematical treatment of the problem by treating each partition independently, i.e. in "isolation". This can be done by exploiting the proposition that, in steady state, the probability of switching from partition P_j to partition P_k equals the probability of switching from partition P_k to P_j. In other words, the probability of entering a partition must be equal to the probability of leaving that partition. Once each partition is solved in "isolation", the original system can be rebuilt by introducing the concept of "partition probability", i.e., the probability for the system being in one and only one of the three partitions at a given time instant. This approach leads to the exact closed-form solution for the stationary probability distribution of the system states. Thus, by applying the isolation technique, a set of analytical formulas can be derived to express the probability that the BESS is at a given charge level.

3. Conclusion

This paper presented a stochastic model for BESS that captures the uncertainty in renewable generation and load demand. By solving the model analytically, we obtained a closed-form solution for the stationary probability distribution of the state of the system. This detailed description of the behavior of the energy storage system could serve as a foundation for optimization or as a benchmark for evaluating heuristic management strategies. Future research directions include extending the model to multiple energy storage devices, incorporating battery degradation mechanisms, and integrating the model with real-time energy management optimization frameworks.

Acknowledgments

This study was carried out within the MICS (Made in Italy – Circular and Sustainable) Extended Partnership and received funding from the European Union Next-GeneraFonEU (PIANO NAZIONALE DI RIPRESA E RESILIENZA (PNRR) – MISSIONE 4 COMPONENTE 2, INVESTIMENTO 1.3 – D.D. 1551.11-10-2022, PE00000004). This manuscript reflects only the authors' views and opinions, neither the European Union nor the European Commission can be considered responsible for them.

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