# Modelling and Analysis of A Remanufacturing System with Different Quality Returned Material and Finished Goods

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Effective decision-making in a circular value chain is pivotal for achieving environmental sustainability and cost efficiency. This study delves into the complexities of returned material purchasing and refurbishing decisions within a circular value chain. Specifically, it focuses on a refurbisher operating in a make-to-stock fashion, considering the refurbishment of first- and second-quality returned materials into corresponding finished goods. The refurbisher must make critical decisions regarding purchasing returned materials and determining the refurbishing processes, encompassing five distinct refurbishing options and two purchasing decisions. The optimal control problem is formulated as a Markovian Decision Process, and a linear programming approach is employed to derive the optimal policy. Analysis of the optimal policy for a wide range of system parameters shows that the optimal refurbishment policy is driven by the per-unit profits for different refurbishment options and also by the returned material, demand rates, and production times for different conversion options. Hence, this result emphasizes the necessity of a comprehensive model capturing the dynamics of material and demand flows, together with the prices and costs, for making the right refurbishment and purchasing decisions in a circular value chain.

Key words: Circular Economy; Refurbishment; Markov Decision Process

### 1. Introduction

Product reuse is not a recent phenomenon but has gained significant traction in recent decades. Product reuse is now recognized as a profitable and sustainable strategy for many companies. Motivations for adopting such reusing strategies include economic, legislative, and environmental factors. For such systems, making the right decisions is more complicated due to the differences in the quality of returned material flows and refurbished products, different demand patterns for refurbished products, and differences in the purchasing and refurbishing costs, among others. Extensive systematic overviews of such systems and possible challenges related to their operational decisions have been proposed by Dekker (2004) and Khan et al. (2021).

Motivated by a business model of packing returned battery cells of different remaining capacities into different quality refurbished packs intended for different uses, here, we consider a discretematerial flow remanufacturing system in which a single facility with a limited production capacity operates in a make-to-stock manner to fulfill the demand for a single product's high- and lowquality versions. The system is portrayed in Figure 1 and its details are given in the next section. In the relevant literature, Vercraene et al. (2014) and Nadar et al. (2023) are the most pertinent studies to our work. However, our work differs from these studies by using a stochastic modeling approach to address an integrated purchasing and remanufacturing decision for a remanufacturing system operating in a make-to-stock fashion.



Figure 1. Purchasing, production and remanufacturing decisions for a system with two returned material and two finished good quality levels

## 2. Problem Definition

The system has two different forms of inventory to perform its operations: (i) returned material and (ii) finished goods inventories. The remanufacturer observes the first- and second-quality returned material levels directly, as well as the inventory levels of first- and second-quality finished goods.

The arrival processes of the returned material types are modeled as two independent Poisson processes, i.e., the inter-arrival times of the first- and second-quality returned materials are considered to be independent of each other, and they are exponentially distributed with rates  $\delta_1$  and  $\delta_1$ , respectively. Each time a returned material unit at any quality level arrives at the system, the remanufacturer decides whether to buy it. If the remanufacturer decides to purchase a first-quality (second-quality) returned material that has arrived at the system, she pays a purchasing cost of  $p_1$  ( $p_2$ ). The costs of keeping one unit of stock in the first- and second-quality returned material buffers are assumed to be  $h_1$  and  $h_2$ , respectively.

The remanufacturer makes five different types of decisions related to the production process. She may decide (*i*) not to produce, (*ii*) to produce a first-quality finished product from a first-quality returned material, (*iii*) to produce a first-quality finished product from a second-quality returned material, (*iv*) to produce a second-quality finished product from a first-quality returned material, and (*v*) to produce a second-quality finished product from a second-quality returned material. The system has a limited production capacity, processing one returned material unit at a time. The system's returned material processing times follow an exponential distribution whose rate depends on the remanufacturer's production decision. The average time being spent to process a unit of returned material of quality *r* to a unit of finished good of quality *f* is denoted by  $1/\mu_{r,f}$  where  $r, f \in \{1, 2\}$ . Furthermore, the processing cost of a unit of returned material is  $c_{r,f}$ .

Two customer types arrive at the system, demanding the same finished goods but at different quality levels. The inter-arrival times of the customers demanding first- and second-quality finished goods are considered to be independent of each other, and they are exponentially distributed with rates  $\lambda_1$  and  $\lambda_2$ , respectively. The manufacturer earns  $s_1$  and  $s_2$  for each sales transaction of first-

and second-quality finished goods, respectively. It is assumed that  $s_1 > s_2$ . We also assume a stockout-based substitution. Suppose a demand for a high-quality (low-quality) product arrives. In such a case, if the high-quality (low-quality) stock is empty and a low-quality (high-quality) product is available,  $\varphi_{1,2}$  ( $\varphi_{2,1}$ ) percent of the customers substitute the high-quality (low-quality) product with a low-quality (high-quality) one. Additionally, the costs of keeping one unit of stock in the firstand second-quality finished goods buffers are assumed to be  $k_1$  and  $k_2$ , respectively.

Given that the remanufacturer fully observes her inventory levels and market prices for her finished goods, she attempts to determine the optimal returned material purchasing and refurbishment strategies that maximize her average profit over an infinite planning horizon.

The inter-event times in the model are considered to be exponentially distributed random variables; therefore, the system dynamics can be characterized as a continuous time Markov Chain. This enables us to model our problem as a Markovian decision process and derive its optimal policies using well-known solution methods. In this study, we employ the Linear Programming (LP) approach, among several other alternatives, to obtain the optimal policy for the system. Using the LP approach empowers us to leverage state-of-the-art optimization software packages, enabling a more expedient problem resolution. As a result, the identification of the optimal policies can be achieved in significantly less time compared to classical methods like value/policy iterations. For the sake of brevity, we refrain from presenting our Markov Decision Process model and Linear Programming formulation here.

## 3. **Results**

Using the LP formulation we devised, we conducted extensive numerical experiments to examine the factors that affect the optimal policy structure and various performance measures. We show that the optimal refurbishment policy depends on the sales prices of different quality products, costs of different quality returned materials, production costs for different refurbishment conversions, the returned material, demand rates, and production times. Therefore, the dynamic stochastic model presented in this study allows managing the refurbishment operations more effectively. Furthermore, following the event-based dynamic programming framework, we analytically show that the optimal purchasing and conversion decisions are of state-dependent threshold policies.

### Acknowledgments

European Union's Horizon 2020 Research and Innovation Programme [AutoTwin project under grant agreement No. 101092021].

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